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Evaluation of Groundwater Replenishment Coefficients From the Record of a Borehole Penetrating the Unsaturated Zone

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The coefficients of groundwater replenishment from surficial sources are the most critical parameters in assessing the exploitable yield of an aquifer. So far, the methods used for estimating these coefficients failed to yield plausible results, either because of crude models of the system or because of a simultaneous estimation of many parameters requiring long and diverse data records. This paper proposes a method for obtaining the in situ chronological sequence of the coefficients of replenishment at a given site, independently from the estimates of other parameters. It is based on the profiles of water content and concentrations of tritium and chloride in a single borehole penetrating the unsaturated zone. A combination of these data with the historical record of their inputs at the surface of the ground enables dating of the moisture content in each depth interval. This yields a series of seasonal replenishment coefficients from both natural and anthropogenic sources. The application of the method is illustrated at two sites in the central and southern parts of Israel.

1. INTRODUCTION

Forecast of the piezometric head in groundwater is made by utilizing groundwater flow models that are either analytic or numerical. The application of any of such models to an actual field problem requires an estimation of the model parameters, a task referred to as calibration of the model. Obviously, an estimation of the error of prediction also requires an assessment of the estimation error of the individual parameters.

Two types of parameter estimation methods are commonly used: local methods and regional methods. A review of some of these methods is given by *Bear* [1979].

The local methods are usually based on the analysis of data from a single groundwater well. They include pump tests, slug tests, and hydrograph analysis. The interpretation of the data is usually based on an ideal model of the system, such as homogeneity and isotropy of the aquifer, homogeneous groundwater, instantaneous response, etc. Moreover, the test results often represent characteristics of the well and of its immediate vicinity rather than those of the aquifer [*Dax*, 1987].

The regional methods deal with solving the groundwater flow equation in a region with respect to the unknown parameters (often referred to as solution of the inverse problem) on the basis of a historical record of data pertinent to all components of the groundwater balance in the region (e.g., precipitation, water consumption in the area overlying the aquifer, water levels in wells, pumpage, and artificial recharge). The techniques of solution are either indirect (e.g., via simulation combined with adjustment of the parameters by trial and error) or direct optimization, employing various objective functions [*Bachmat et al.*, 1980].

Regional methods have gained widespread popularity thanks to the rapid development of numerical techniques and computational capacity. However, the results of regional calibration methods have not yet proven to be satisfactory for the practicing hydrologist. Indeed, often the regional methods yield parameter estimates beyond the range of hydrologic plausibility e.g., negative coefficient of replenishment and storativity exceeding unity [*Neuman*, 1973]. On the other hand, constrained optimization (i.e., optimization with prior imposed constraints on parameter estimates) often leads to estimates coinciding with the constraints and thus render the data record useless [*Bachmat and Dax*, 1979].

Another drawback of the regional calibration methods of multicell aquifers is the need to solve simultaneously a large system of equations for many parameters. Calibration of such a model requires a sufficiently long record of data pertinent to all components of the groundwater balance, which is often difficult to find. In this way an error because of uncertainty in one category of data can affect errors in parameters pertinent to other data categories.

The most critical parameters in assessing the exploitable groundwater yield of an aquifer are the coefficients of replenishment from surficial sources, both natural and anthropogenic. As indicated above, regional calibration methods usually fail to provide estimates of these parameters which are both hydrologically plausible and have a sufficiently small estimation error. As to the local methods listed above, only one of them, namely hydrograph analysis, provides estimates of replenishment coefficients and this only in combination with other parameters. The ratio of chloride content in well water and in rainwater, which is sometimes used as an estimate of the coefficient of natural replenishment, is based on idealized conditions which rarely prevail in nature.

Viswanthan [1984] evaluates three methods for estimating natural recharge rates of unconfined aquifers: experimental (e.g., by lysimeters), mass balance methods and time series models. According to Viswanthan, the experimental methods are expensive, time consuming, and restrictive because

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of distortion of the natural conditions. The mass balance methods which are based on solving the differential groundwater flow equations require some knowledge of other parameters. Finally, time series analysis, which estimates the recharge from the relationship between a given record of rainfall and a series of groundwater level readings in a borehole, is based on an oversimplified model of a groundwater balance and still requires some knowledge of the storativity.

The objective of this paper is to propose a method for obtaining the statistical distribution of the coefficients of replenishment, independently from estimates of other parameters, on the basis of data derived from a single well penetrating the unsaturated zone, in conjunction with a record of inputs applied at the surface.

2. DESCRIPTION OF THE METHOD

Consider a small-diameter borehole drilled by a dry method through the unsaturated zone down to the groundwater table of a phreatic aquifer. While drilling, soil and sediment samples are taken from the borehole at small depth intervals. Water extracted from each sample is analyzed to determine the moisture content θ and the concentration of a tracer C. The procedure of sampling, analysis, and the presentation of the obtained profiles of θ and C are described by *Gvirtzman and Magaritz* [1986]. Given these profiles, the next step is dating the moisture contained in each depth interval. A precondition for such dating is availability of data on variations of tracer concentration in the water applied to the surface in the past. The presence of such variations exhibits itself in the form of peaks and troughs in the profile of the tracer. The sequence of the peaks and troughs is related to a chronological sequence of years by matching the tracer concentration in the applied water to that found in the tracer profile.

The accuracy of the dating depends on the degree to which the tracer moves with the water molecules. Thus isotopes of water such as tritium, deuterium, and O^{18} are the best tracers as they are a part of the water molecule. Practically, tritium was found to be most suitable because its concentration varies significantly from one source of water to another. An additional advantage in using tritium as a tracer is the drastic increase of its concentration in precipitation during the period of the thermonuclear tests and its decrease afterward. Conservative tracers, other than isotopes of water, such as chloride may also be used, although with less accuracy, owing to deviations of the velocity of such tracers from that of the water, caused by anion exclusion [*Gvirtzman et al.*, 1986].

Given the record of water application to the surface of the ground (in terms of source and volume of water) during a certain period of time, and given the corresponding chronological sequence of moisture content in the unsaturated zone, the ratio of their volumes for each time interval yields an estimate of the coefficient of groundwater replenishment during that time interval. On the other hand, regression of the series of volumes in the unsaturated zone on the corresponding series of input volumes above the surface provides an estimate of the coefficient of replenishment over the entire period of record.

3. THE MATHEMATICAL MODEL

The mathematical model used to estimate the coefficients of groundwater replenishment is based on the following assumptions: (1) Water entering the subsurface moves downward vertically only, (2) there are no sources or sinks of water within the dating zone which extends from the depth affected by evapotranspiration down to the top of the capillary fringe, and (3) the specific discharge of water does not exceed the vertical hydraulic conductivity corresponding to the moisture content throughout the dating zone.

Let $(\Delta Z)_i$ be the depth interval of a sample containing water from a given source that, according to the dating, was applied at the surface of the ground during the ith interval of time. If θ_i is the moisture content of that sample, then the volume of water in it per unit horizontal area is $\theta_i(\Delta Z)_i$. According to the historic record, the volume of water supplied to the surface per unit area in the ith time interval is A_i. The ratio

$$\alpha_i = \theta_i(\Delta Z)_i/A_i \tag{1}$$

is the fraction of the water applied during the ith interval of time, which will reach the groundwater. This fraction is referred to as the coefficient of replenishment related to the ith time interval.

Replenishment from different sources of water can be distinguished by indexing the source. Let $N_i^{(k)}$ denote the replenishment of groundwater (in terms of volume per unit area) from a given source of water (index k) that was applied to the surface of the ground during the ith time interval. By equation (1),

$$N_i^{(k)} \equiv \alpha_i^{(k)} A_i^{(k)} = \theta_i(\Delta Z)_i|_{(k)} \tag{2}$$

Groundwater flow models which take into account uncertainty commonly express the relationship between $N_i^{(k)}$ and its corresponding surface source in the form

$$N_i^{(k)} = \alpha^{(k)} A_i^{(k)} + \varepsilon_i \tag{3}$$

or

$$N_i^{(k)} = \alpha^{(k)} A_i^{(k)} + \beta^{(k)} + u_i \tag{4}$$

where $\alpha^{(k)}$ and $\beta^{(k)}$ are constants related to the kth source of water, and ε_i or u_i are random residuals. Given either one of the above models, any forecast of replenishment in the future for a given intensity of a source of supply at the surface is made by using predictors of the type

$$\hat{N}^{(k)} = \hat{\alpha}^{(k)} A^{(k)} \tag{5}$$

or

$$\hat{N}^{(k)} = \hat{\alpha}^{(k)} A^{(k)} + \hat{\beta}^{(k)} \tag{6}$$

where $\hat{\alpha}^{(k)}$ and $\hat{\beta}^{(k)}$ are the best estimators of $\alpha^{(k)}$ and $\beta^{(k)}$ subject to a preselected criterion of optimality. Thus, for example, the best linear unbiased estimators (BLUE) of the above parameters, based on the model of ordinary least squares and a sample of n observations, are given by *Maddala* [1977]:

$$\begin{aligned} \hat{\alpha}^{(k)} &= \frac{\sum N_i^{(k)} A_i^{(k)}}{\sum (A_i^{(k)})^2} = \frac{\sum \alpha_i^{(k)} (A_i^{(k)})^2}{\sum (A_i^{(k)})^2} \\ \sigma_{\hat{\alpha}^{(k)}}^2 &\equiv Var\ \hat{\alpha}^{(k)} = \frac{\sigma_\varepsilon^2}{\sum (A_i^{(k)})^2} \end{aligned} \tag{7}$$

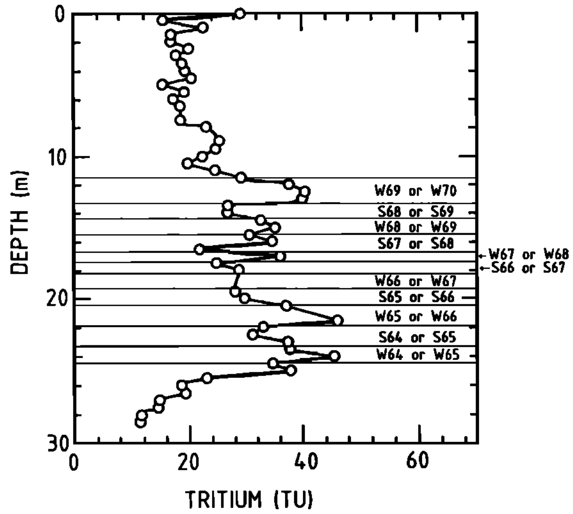

Fig. 1. Seasonal dating of depth intervals and their moisture content in the WT-2 well (Coastal Plain), based on the tritium profile. W denotes winter, S denotes summer, and numbers beside W or S denote years.

for the predictor (5), or

$$
\begin{aligned}
\hat{\alpha}^{(k)} &= \frac{\sum \dot{N}_i^{(k)} \dot{A}_i^{(k)}}{\sum (\dot{A}_i^{(k)})^2} = \frac{\sum \alpha_i^{(k)} (\dot{A}_i^{(k)})^2}{\sum (\dot{A}_i^{(k)})^2} \\
\sigma^2_{\hat{\alpha}^{(k)}} &\equiv Var\ \hat{\alpha}^{(k)} = \frac{\sigma_u^2}{\sum (\dot{A}_i^{(k)})^2} \\
\hat{\beta}^{(k)} &= \bar{N}^{(k)} - \hat{\alpha}^{(k)} \bar{A}^{(k)} \\
\sigma^2_{\hat{\beta}^{(k)}} &\equiv Var\ \hat{\beta}^{(k)} = \sigma_u^2 \left[\frac{1}{n} + \frac{(\bar{A}^{(k)})^2}{\sum (\dot{A}_i^{(k)})^2} \right]
\end{aligned}
\qquad (8)
$$

for the predictor (6); where σ_ε^2 and σ_u^2 are variances of the residuals, and

$$\sum \equiv \sum_{i=1}^{n}$$

$$(\dot{\ }) = (\ ) - (\bar{\ })$$

$$(\bar{\ }) = \frac{1}{n} \sum_{i=1}^{n} (\ )_i$$

In practice, $Var\ \hat{\alpha}^{(k)}$ and $Var\ \hat{\beta}(k)$ are estimated by replacing σ_ε^2 or σ_u^2 with $RSS/(n-1)$ or $RSS/(n-2)$, respectively, where $RSS = \Sigma\ (N_i^{(k)} - \hat{N}_i^{(k)})^2$. Confidence intervals of the parameter estimates are obtained by assigning distribution functions to ε and u, respectively.

In practice, water extracted from a sample at depth $(\Delta Z)_i$ may include two components: mobile water and immobile water. According to the assumptions of the model described above, only the mobile water belongs to the water supplied to the surface during the ith time interval.

An alternative model for relating replenishment to supply of water at the surface is the model

$$N^{(k)} = \alpha_*^{(k)} A^{(k)} \qquad (9)$$

where $\alpha_*^{(k)}$ is considered to be a random variable, as opposed to $\alpha^{(k)}$ of equation (3) which is considered to be a constant. In this case, one may use a predictor of replenishment of the type

$$\hat{N}^{(k)} = E\hat{\alpha}_*^{(k)} A^{(k)} \qquad (10)$$

where

$$E\hat{\alpha}_*^{(k)} \equiv \bar{\alpha}_*^{(k)} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i^{(k)} = \frac{1}{n} \sum_{i=1}^{n} \frac{N_i^{(k)}}{A_i^{(k)}} \qquad (11)$$

is an unbiased estimator of $E\alpha_*^{(k)}$.

4. Application and Results

The proposed method was applied in interpreting field data records obtained from a small diameter borehole, WT-2, drilled through the unsaturated zone in the Coastal Plain of Israel and an excavation penetrating the unsaturated zone at the Omer site in the Northern Negev. Details pertaining to the sampling, analysis, and dating of the moisture content profiles of these locations, using tritium as a tracer, are described by *Gvirtzman and Magaritz* [1986] and *Gvirtzman et al.* [1986].

The WT-2 well is located in a cultivated area receiving water from two sources: rainfall in the winter season and treated sewage in summer. The dating of peaks in the tritium profile along the sediment column served as a basis for assigning depth intervals, and hence also their moisture content, to years and seasons, as presented in Figure 1. As explained by *Gvirtzman et al.* [1986], two alternative datings of the tritium peaks were possible. Figure 1 shows these two alternatives. Another possibility of dating the profile of moisture content using chloride as a tracer is presented in Figure 2.

The Omer excavation is also located in a cultivated area which receives rainfall in winters and is irrigated with local

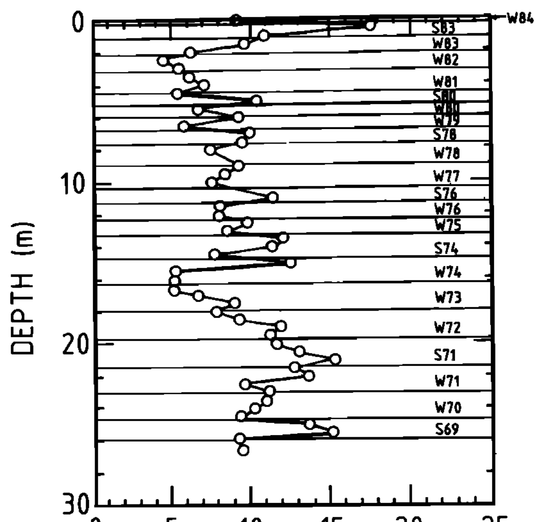

Fig. 2. Seasonal dating of depth intervals and their moisture content in the WT-2 well (Coastal Plain), based on the chloride profile. W denotes winter, S denotes summer, and numbers beside W or S denote years.

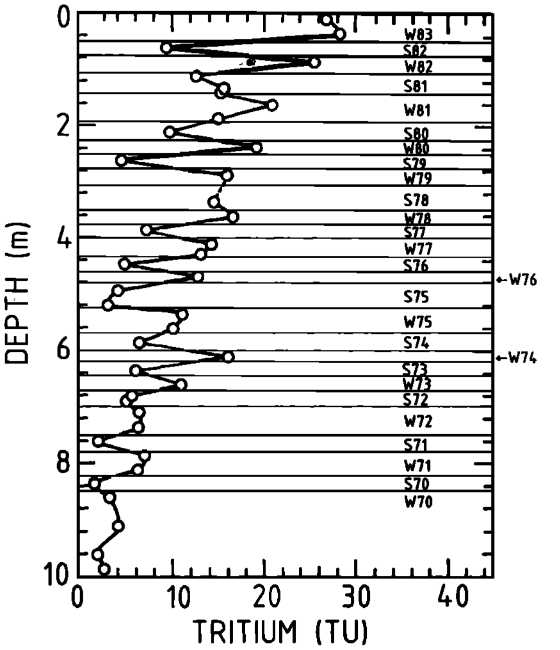

Fig. 3. Seasonal dating of depth intervals and their moisture content at Omer (Northern Negev) based on the tritium profile. W denotes winter, S denotes summer, and numbers beside W or S denote years.

groundwater in summers. Seasonal dating of depth intervals at Omer are presented in Figure 3.

Data containing the chronological sequence of water content in the sediment column at each of the two sites, the corresponding amounts of water applied at the surface, and the seasonal coefficients of replenishment are presented in Tables 1, 2, and 3. These data were used to obtain BLUE estimates of replenishment coefficients from rainfall (in winter) and from irrigation (in summer), employing single linear regression with the predictors (5) and (6), using both tritium and chloride as tracers. The results are summarized in Table 4. Table 4 also holds the sample average of α and its standard deviation.

5. Discussion

A predictor of replenishment with no constant is preferable to that with a constant. Indeed, as shown in Table 4, the predictor with no constant has in all cases a correlation coefficient close to 1, while the predictor with a constant is less than 0.5. Also, the estimated constant is always positive and has no physical meaning. In the absence of water application there can be no replenishment (i.e., 0). On the other hand, if there was a threshold value of application which does not contribute to replenishment, then should be negative.

It is also worth noting that the sample average value of is slightly higher than the BLUE estimator , owing to the different weighting coefficients (the BLUE estimates assign a higher weight to higher applications). Accordingly, the BLUE estimates reflect the fact that the higher the level of application, the smaller the error in estimating the coefficient of replenishment.

A remarkable phenomenon is the difference between the coefficient of replenishment in the winter season versus that in summer. At the WT-2 site the coefficient of replenishment in winter is lower than that in summer, whereas at Omer it is quite the contrary. It can be explained by the differences in climate and agrotechnical methods.

The evaluation of replenishment coefficients at the site of the WT-2 well, using tritium as a tracer, is based on data from the sand portion of the sediment column only (Table 4). Due to the relatively poor distinction between seasonal peaks in the clay–loamy part of the sediment column it was impossible to evaluate replenishment coefficients on a seasonal basis in that part.

The estimated values of α, based on chloride as a tracer, differ from those based on tritium. This can be explained by the fact that chloride ions move faster than water molecules [*Gvirtzman et al.*, 1986], which produces an error in the dating. The error may cause either an overestimate of α (as in our case) or an underestimate, depending on the chronological sequence of the application rates.

The values of the computed coefficient of replenishment in the clay–loamy part of the sediment column of WT-2 well, based on the chloride profile (Table 2), yield an average value of $\bar{\alpha} = 41.7\%$, while the average value of α in the sandy part of the column is only 19.4%. This difference can be explained by the fact that the clay–loamy part contains water which does not hold moving Cl^- ions but was taken into account in the estimation $\bar{\alpha}_{clay}$. Since the rates of application in the years corresponding to the sandy part and those of the

TABLE 1. Chronological Sequence of Water Content in the Sand Sediment Column of the WT-2 Well and the Corresponding Amount of Water Applied at the Surface, Based on the Tritium Profile (Figure 1)

Season,* Year	Depth Interval, m	Profile Water N, mm	Applied Water A, mm	Replenishment α, %
W70/1 or W69	11.5–13.3	126	1052 or 806	12 or 21
S69 or S68	13.3–14.3	70	450	16
W69 or W68	14.3–15.6	91	806 or 429	11 or 21
S68 or S67	15.6–16.6	70	450	16
W68 or W67	16.6–17.3	49	429 or 741	11 or 7
S67 or S66	17.3–18.2	63	450	14
W67 or W66	18.2–19.3	77	741 or 415	10 or 19
S66 or S65	19.3–20.4	77	450	17
W66 or W65	20.4–21.9	105	415 or 729	25 or 14
S65 or S64	21.9–23.3	98	450	22
W65 or W64	23.3–24.4	77	729 or 576	11 or 13

*W denotes winter; S denotes summer.

TABLE 2. Chronological Sequence of Water Content in the Sediment Column of the WT-2 Well and the Corresponding Amount of Water Applied at the Surface, Based on the Chloride Profile (Figure 2)

Season,* Year	Depth Interval, m	Lithology	Profile Water N, mm	Applied Water A, mm	Replenishment α, %
W84	0.0–0.2	Clay loam	43	161	27
S83	0.2–1.0	Clay loam	174	450	39
W83	1.0–2.1	Clay loam	239	605	40
W82	2.1–3.1	Clay loam	217	483	45
W81	3.1–4.5	Clay loam	304	508	60
S80	4.5–5.3	Clay loam	174	450	39
W80	5.3–6.0	Clay loam	152	640	24
W79	6.0–6.8	Clay loam	174	338	51
S78	6.8–7.7	Clay loam	195	450	43
W78	7.7–8.9	Clay loam	260	482	54
W77	8.9–10.3	Clay loam	260	693	37
S76	10.3–11.4	Sand	77	450	17
W76	11.4–12.2	Sand	56	457	12
W75	12.2–13.3	Sand	77	582	13
S74	13.3–14.8	Sand	105	450	23
W74	14.8–16.4	Sand	112	704	16
W73	16.4–18.0	Sand	112	462	24
W72	18.0–19.8	Sand	126	712	18
S71	19.8–21.4	Sand	112	450	25
W71	21.4–23.0	Sand	112	567	20
W70	23.0–24.8	Sand	126	484	26
S69	24.8–26.0	Sand	84	450	19

*W denotes winter; S denotes summer.

clay–loamy part are not significantly different, we can present the ratio of the computed averages of α in the form:

$$\frac{\bar{\alpha}_{\text{sand}}}{\bar{\alpha}_{\text{clay}}} = \frac{\bar{N}_{\text{sand}}}{\bar{N}_{\text{clay}}} = \frac{\bar{N}_{(\text{sand–mobile})}}{\bar{N}_{(\text{clay–mobile}} + \bar{N}_{(\text{clay–stagnant})}} \qquad (12)$$

where the subscripts "mobile" and "stagnant" refer to volumes of water which hold moving Cl^- ions and are excluded of them, respectively. Substitution of the above values of $\bar{\alpha}$, given that $\bar{N}_{(\text{sand–mobile})} = \bar{N}_{(\text{clay–mobile})}$, owing to the conservation of the chloride-carrying water, yields

$$\frac{N_{\text{stagnant}}}{N_{\text{stagnant}} + N_{\text{mobile}}} = 0.53$$

TABLE 3. Chronological Sequence of Water Content in the Sediment Column at Omer and the Corresponding Amount of Water Applied at the Surface, Based on the Tritium Profile (Figure 3)

Season, Year	Depth Interval, m	Profile Water N, mm	Mobile Water, mm	Applied Water A, mm	Replenishment* α, %
W83	0.0–0.5	90	55	274	20
S82	0.5–0.7	45	26	650	4
W82	0.7–1.1	52	31	218	14
S81	1.1–1.4	70	39	650	6
W81	1.4–1.9	98	56	223	25
S80	1.9–2.2	69	39	650	6
W80	2.2–2.5	50	28	311	9
S79	2.5–2.7	45	26	650	4
W79	2.7–3.1	52	28	154	18
S78	3.1–3.5	69	39	650	6
W78	3.5–3.7	41	21	109	19
S77	3.7–4.0	45	19	650	3
W77	4.0–4.3	62	31	171	18
S76	4.3–4.6	51	26	650	4
W76	4.6–4.8	37	17	154	11
S75	4.8–5.2	73	32	650	5
W75	5.2–5.7	72	30	199	15
S74	5.7–6.0	48	19	650	3
W74	6.0–6.2	34	14	289	5
S73	6.2–6.4	36	13	650	2
W73	6.4–6.6	46	18	166	11
S72	6.6–7.0	65	26	650	4
W72	7.0–7.5	103	36	330	11
S71	7.5–7.8	61	19	650	3
W71	7.8–8.2	94	33	233	14
S70	8.2–8.5	70	26	650	4

*Ratio of mobile water to the applied water.

TABLE 4. Statistical Analysis of the Replinishment Coefficients*

Site	Tracer	Season	Source of Water	Number of Data	$\bar{\alpha}$	$\sigma_{\bar{\alpha}}$	Predictor	Correlation Coefficient, r	$\hat{\alpha}$	$\sigma_{\hat{\alpha}}$	$\hat{\beta}$	$\sigma_{\hat{\beta}}$
WT-2	tritium	winter	rainfall	12	0.146	0.016	$\hat{N} = \hat{\alpha}A$	0.958	0.127	0.012	0	0
WT-2	tritium	winter	rainfall	12			$\hat{N} = \hat{\alpha}A + \hat{\beta}$	0.444	0.055	0.035	51.3	24.1
WT-2	tritium	summer	irrigation	5	0.170	0.013	$\hat{N} = \hat{\alpha}A$	0.987	0.168	0.013	0	0
WT-2	chloride	winter	rainfall	7	0.184	0.020	$\hat{N} = \hat{\alpha}A$	0.973	0.179	0.017	0	0
WT-2	chloride	winter	rainfall	7			$\hat{N} = \hat{\alpha}A + \hat{\beta}$	0.378	0.092	0.101	50.7	58.3
WT-2	chloride	summer	irrigation	4	0.210	0.018	$\hat{N} = \hat{\alpha}A$	0.988	0.210	0.019	0	0
Omer	tritium	winter	rainfall	13	0.146	0.015	$\hat{N} = \hat{\alpha}A$	0.924	0.134	0.016	0	0
Omer	tritium	winter	rainfall	13			$\hat{N} = \hat{\alpha}A + \hat{\beta}$	0.329	0.063	0.054	17.0	12.3
Omer	tritium	summer	irrigation	13	0.042	0.004	$\hat{N} = \hat{\alpha}A$	0.958	0.041	0.004	0	0

*Data of WT-2 site pertain only to the sandy part of the sediment column. Data of Omer site pertain to the entire sediment column.

i.e., the Cl^- excluded portion of the profile water in the clay–loamy part is a half of the total water. A similar result was obtained by *Gvirtzman et al.* [1986].

Finally, it is worthwhile noting that the method proposed in this paper is restricted to fields receiving at least two isotopically distinct sources of recharge water. Also, a record of irrigation amounts during the relevant number of years is necessary for applying this method.

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