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KOZENY-CARMAN RELATION FOR A MEDIUM WITH TAPERED CRACKS

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Abstract. We examine the permeability of a medium with thin tapered cracks to a single-phase fluid flow in the presence of immobile matter which is accumulated in the tips of cracks. The original Kozeny-Carman relation shows an increase in permeability of such a material relative to the case when tips are free of accumulated matter. To resolve this paradox we introduce a corrected version of the Kozeny-Carman relation for the case when the shape of a crack cross-section can be described by a power law. This class of crack shapes includes the important cases of triangular cracks and space between two contacting circular grains. The revised relation includes the original porosity Φ and specific surface area S of the material without accumulated matter as well as the degree of filling a crack space by accumulated matter Z. The permeability is proportional to Φ^3 and S^{-2}, and decreases with increasing Z.

Introduction

The Kozeny-Carman formula has been shown to be a successful instrument relating permeability and other measurable properties of a porous material. Typically this formula can be derived using the flow model in which the complicated flow network through the pore space of a material is replaced by a single representative conduit. Walsh and Brace (1984) presented this relation in the following form:

$$k = \frac{1}{k_s}\frac{\Phi^3}{S^2T^2}, \tag{1}$$

where k is permeability; Φ is porosity; S is specific surface area (the surface area per unit volume); T is tortuosity; and k_s is the pore shape coefficient. Parameters Φ and S can be introduced into the Kozeny-Carman relation through the concept of the hydraulic radius that is defined as the ratio of the volume of a conduit to its wetted area. Tortuosity T can be defined as the actual fluid path length relative to the apparent path length. The factor k_s is a weak function of the pore shape of the conduit cross-section. Typically $k_s = 2$ for circular tubes and $k_s = 3$ for flat cracks. Berryman and Blair (1987) showed that equation (1) is expected to be a reasonable approximation when the distribution of pore sizes and shapes is narrow.

In this paper we focus on the applicability of equation (1) to a medium with thin tapered cracks with immobile matter accumulated in the crack tips. This accumulation

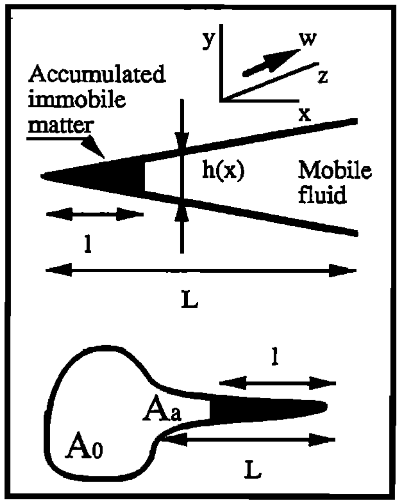

Figure 1. Thin tapered pore and pore with tapered appendix with accumulated immobile matter.

may occur due to mineral deposition in pores or precipitation of salts. It may also be caused by a trapped immobile wetting phase. We assume that a mobile phase filtrates through pore space free from accumulated matter and that all free pore space is involved in the flow process. The cross-section of a representative conduit, one typical of the sample of a medium as a whole, is shown in Figure 1.

The tortuosity of the pore space occupied by the mobile fluid may change depending on the amount of accumulated immobile matter. Yet, in the case under consideration it is natural to assume that all cross-sections of the representative conduit are equally reduced by the accumulated matter. This means that the process of immobile phase accumulation will not change the actual fluid path length and T can reasonably be considered the same for different amounts of accumulated immobile matter.

Thus, the medium with accumulated immobile matter can be considered, regarding the filtration of the mobile phase, as a porous material which differs from the original (without accumulated matter) material only by the addition of the solid. The tortuosity of such a material is the same as that of the original material, but porosity and specific surface area will be changed by the reduction of the conduit opening.

The process of accumulation of the immobile matter in tapered tips at its initial stage results in small reduction of porosity and significant reduction of specific surface area. Thus, if T is constant, formula (1) will give an increase in permeability where a reduction might reasonably be expected. This computed artifact is similar to the effect of surface roughness of a circular conduit. Berryman and

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Blair (1987) show that the surface area to be used in the Kozeny-Carman relation is that of a smoothed version of the true void/solid interface.

In this paper we derive a corrected version of the Kozeny-Carman equation for the case when the shape of the representative conduit's cross-section can be described by a power law

$$h(x) = cx^r, \quad (2)$$

where $h(x)$ is the thickness of the crack as a function of the x coordinate (Figure 1); c and r are constants. This general profile description includes two cases of practical importance: (1) triangular pores (r=1) and (2) pores formed by contacting circular grains (r=2). A view of "tubular" pores with triangular and parabolic cross-sections is given in micrographs by Bernabe, Evans and Brace (1982). Our corrected version of the Kozeny-Carman equation includes the pore-shape parameter r and the degree of filling the pore space by immobile matter Z. In many cases practical computations are based on a given value of Z. As, for example, when the permeability to gas has to be found in the presence of trapped immobile oil at a known oil saturation. The value of r can be found from general considerations (assuming, for example, that cracks are triangular for a certain rock type) or from image processing methods [Berryman and Blair, 1987].

In addition we introduce a corrected version of the Kozeny-Carman equation for a medium with pores having thin tapered appendixes. In this case we have to take into account several parameters in order to describe pore geometry. These parameters may be difficult to measure or estimate. Still, this formula can be useful in some applications when permeability has to be found as a function of a given pore geometry.

Flow Through a Thin Crack

The equation of two-dimensional slow viscous flow in a representative conduit (Figure 1) is [Langlois, 1964]:

$$w_{xx} + w_{yy} = P_z/\mu, \quad (3)$$

where w is fluid velocity in the z direction, perpendicular to the conduit's cross-section; the x and y axes are placed in the conduit's cross-section; P is pressure; and μ is fluid viscosity. Lower-case indexes denote partial derivatives.

The approximate solution of equation (3) is:

$$w(x,y) = Gy[h(x) - y]/2, \quad (4)$$

where $G = -P_z/\mu$. This formula was obtained under the assumption that the velocity distribution at every x is approximately identical to that when fluid flows through a slit of equal thickness.

To show the adequacy of formula (4) to describe a flow through a thin conduit we perform the following estimates. Velocity derivatives can be found from (4) as follows: $w_{yy} = -G$; $w_{xx} = Gyh''/2$. Estimating the derivatives of $h(x)$: $y \approx h$, $h' \approx h/L$, $h'' \approx h/L^2$, we arrive at the following expression for w_{xx}: $w_{xx} \approx G(h/L)^2/2$, where L is the length of the conduit's cross-section.

Using this estimate we can conclude that if the crack is thin ($h \ll L$), then $| w_{xx} | \ll | w_{yy} |$, and solution (4) becomes acceptable for equation (3).

Integrating (4) in the y direction from 0 to $h(x)$ we can find fluid discharge q through a column over element dx: $q = G\frac{h^3}{12}dx$. Integrating this expression in the x direction from l to L we find discharge Q of mobile fluid through the conduit:

$$Q = \int_l^L q(x)dx = \frac{G}{12}\int_l^L h^3(x)dx. \quad (5)$$

Here l is the length of the conduit's cross-section occupied by immobile matter (Figure 1).

Permeability to a Mobile Fluid

When the shape of a pore can be expressed by a power law (2) we obtain from (5) the following expression for the mobile fluid discharge:

$$Q(l) = \frac{G}{12}c^3\int_l^L x^{3r}dx = \frac{Gc^3}{12(3r+1)}(L^{3r+1} - l^{3r+1}). \quad (6)$$

Defining the degree of filling the crack space by immobile matter Z as the ratio of pore volume occupied by immobile matter to the whole pore volume we arrive at the following expression for Z:

$$Z = \frac{\int_0^l h(x)dx}{\int_0^L h(x)dx} = (l/L)^{r+1} \quad (7)$$

Using (7) we express l through L and substitute into (6):

$$Q = \frac{c^3GL^{3r+1}}{12(3r+1)}(1 - Z^{\frac{3r+1}{r+1}}).$$

Given that a unit area of rock cross-section contains n representative conduits, the gross discharge of the mobile fluid $\tilde{Q}$ is:

$$\tilde{Q} = nQ = n\frac{c^3GL^{3r+1}}{12(3r+1)}(1 - Z^{\frac{3r+1}{r+1}}). \quad (8)$$

The original porosity Φ of the medium without accumulated immobile matter is:

$$\Phi = n\int_0^L h(x)dx = ncL^{r+1}/(r+1).$$

Employing the assumption that cracks are thin ($h \ll L$) we can use the following approximation for the original specific surface area S of the media without immobile matter: $S \approx 2nL$. Substituting these expressions for Φ and S into (8) we have:

$$\tilde{Q} = \frac{(r+1)^3}{3(3r+1)}\frac{\Phi^3}{S^2}(1 - Z^{\frac{3r+1}{r+1}})G.$$

Defining permeability k as $k = \tilde{Q}/G$ and taking into account the effect of tortuosity (see Introduction) we arrive at the following modification of the Kozeny-Carman formula (1):

$$k = \frac{(r+1)^3}{3(3r+1)}\frac{\Phi^3}{T^2S^2}(1 - Z^{\frac{3r+1}{r+1}}), \quad (9)$$

where Φ is the original porosity, and S is the original specific surface area of a medium at $Z = 0$.

For the case of a triangular-shaped pore ($r = 1$), expression (9) becomes: $k = \frac{2}{3}\frac{\Phi^3}{T^2S^2}(1 - Z^2)$; if cracks are formed by contacting circular grains (r=2): $k = \frac{9}{7}\frac{\Phi^3}{T^2S^2}(1 - Z^{\frac{7}{3}})$;

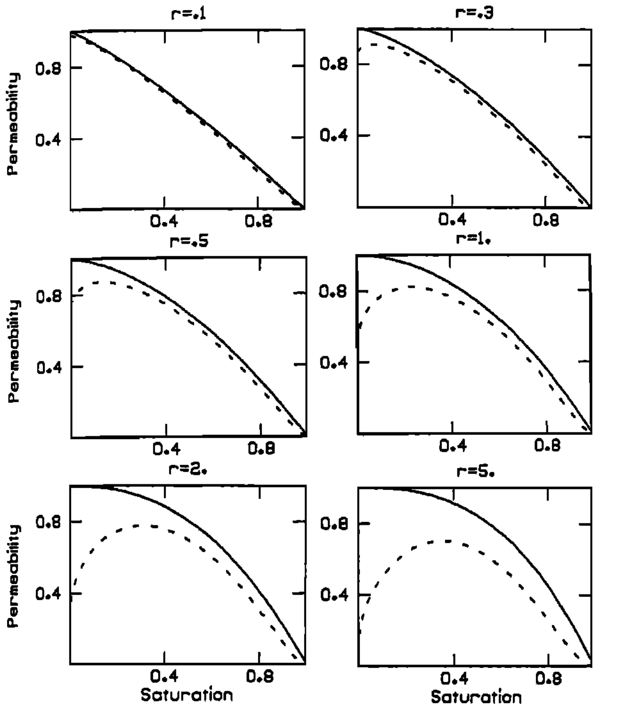

Figure 2. Permeability of medium with tapered cracks using corrected (solid line) and original (dotted line) Kozeny-Carman equations. Permeability was normalized by its value at $Z = 0$.

The relationship between S and permeability as given by formula (9) is plotted in Figure 2 for different crack shapes. These results indicate a dramatic difference between permeabilities calculated using formulas (1) and (9) for highly tapered cracks. In the case of slightly tapered cracks ($r = 0.5$) the difference is small. This difference becomes negligible for the cases $r = 0.3$ and $r = 0.1$ when the shape of cracks approaches the shape of a slit. The original Kozeny-Carman formula is adequate for calculating permeability due to smooth or slightly tapered pores but fails to deal with highly tapered cracks. The corrected version (9) offers a reasonable estimate of k. Permeability decreases with increasing accumulation of immobile matter.

Pore with Tapered Appendixes

The failure of the Kozeny-Carman formula (1) to predict the permeability of media with tapered cracks in the presence of accumulated immobile matter also applies to media with pores having tapered appendixes (Figure 1).

In this case we assume that a representative conduit can be approximately divided into its main part and an appendix (or several appendixes). We also assume that the permeability by the main part k_m is known and it is approximately independent of the fluid flow through the appendix.

If immobile matter is accumulated in the appendix, we can use the results derived above and express the gross mobile fluid discharge through the conduit $\tilde{Q}_g$ as the sum of the discharges through its main part $\tilde{Q}_m$ and through the appendix $\tilde{Q}(l)$.

$$\tilde{Q}_g = \tilde{Q}(l) + \tilde{Q}_m = \frac{(r+1)^3}{3(3r+1)} \frac{\Phi^3}{T^2 S^2}(1 - Z^{\frac{3r+1}{r+1}})G + k_m G,$$

where $k_m = \tilde{Q}_m / G$ is the permeability by the main part of the conduit. Thus, the permeability of a porous medium to the mobile fluid can be approximately calculated as the sum of permeabilities by the main part of the pore and by the appendix:

$$k = k_m + \frac{(r+1)^3}{3(3r+1)} \frac{\Phi^3}{T^2 S^2}(1 - Z^{\frac{3r+1}{r+1}}). \qquad (10)$$

The term Z in this equation denotes the degree of filling the appendix by immobile matter. This parameter can be expressed through the degree of filling the whole pore space Z_0: $Z = Z_0 m$, where $m = A_0/A_a$ is the ratio of the whole pore cross-section area A_0 to the appendix area A_a. The terms Φ and S define the part of porosity and specific surface area related to the appendix. They can be expressed through the porosity Φ_0 and specific surface area S_0 of the porous media: $\Phi = \Phi_0/m$; $S = S_0/n$. Here $n = \Pi_0/\Pi_a$, where Π_0 is the circumference of the conduit, and Π_a is the circumference of the appendix.

Substituting these formulas into (10) we express the permeability in terms of degree of filling by immobile matter, porosity and specific surface area of a porous medium:

$$k = k_m + \frac{(r+1)^3}{3(3r+1)} \frac{\Phi_0^3}{T^2 S_0^2} \frac{n^2}{m^3}[1 - (mZ)^{\frac{3r+1}{r+1}}]. \qquad (11)$$

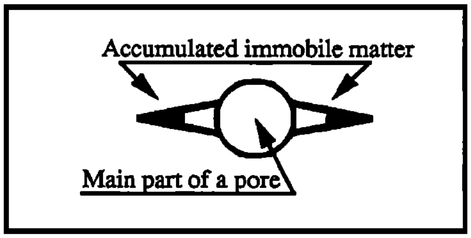

Figure 3. Circular pore with two thin triangular appendixes.

To demonstrate the application of equation (11) we calculated the permeability of material with circular cylindrical pores surrounded by two thin triangular appendixes (Figure 3). The permeability by the main part of the pore k_m was calculated by (1); to include the influence of appendixes we used (11) where $r = 1$. The results of calculations (Figure 4) are compared with the estimates of permeability by the original Kozeny-Carman equation.

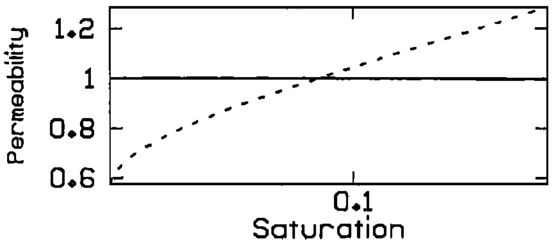

Figure 4. Relative permeability of a medium with circular pores and two thin triangular appendixes using corrected (solid line) and original (dotted line) Kozeny-Carman equations. Permeability was normalized by its value at $Z = 0$.

Permeability to the mobile phase, if computed using (1), increases with increasing accumulation of immobile matter. This artifact indicates the failure of the original Kozeny-Carman equation to predict the permeability of a medium with pores having tapered appendixes as a function of immobile matter accumulation.

Conclusions

We introduced a revised version of the Kozeny-Carman equation to calculate the permeability of a medium with thin tapered cracks (2) or pores having tapered appendixes (11) to a mobile fluid in the presence of immobile matter accumulated in crack tips. Formula (2) includes the pore-shape parameter r and the degree of filling the pore space by the immobile matter Z as well as the original porosity Φ and specific surface area S at $Z = 0$. The permeability is proportional to Φ^3 and S^{-2}, and decreases with increasing Z. Formula (11) includes several additional parameters that may be difficult to measure or estimate. The results of computations indicate a large difference between permeabilities calculated using original and revised formulas for highly tapered pores. In the case of slightly tapered pores the difference is small. The original Kozeny-Carman equation is adequate to calculate the permeability of media with smooth or slightly tapered pores but fails to deal with highly tapered cracks. It shows an increase in permeability of such a material relative to the case when tips are free of accumulated matter. The corrected versions of the Kozeny-Carman equation can be used to avoid this paradox.

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